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1978 J. Phys. A: Math. Gen. 11 L259

(http://iopscience.iop.org/0305-4470/11/11/001)

J. Phys. A: Math. Gen., Vol. 11, No. 11, 1978. Printed in Great Britain

LETTER TO THE EDITOR

'Light ray and particle paths on a rotating disc': a reply to criticisms therein

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Received 9 August 1978

In a recent paper McFarlane and McGill (1978, to be referred to as MM) considered the question of 'Light ray and particle paths on a rotating disc'. Whilst agreeing with the sections of their paper which are similar to the work contained in Jennison (1963, 1964), Ashworth and Jennison (1976), Ashworth and Davies (1977), etc we take this opportunity of replying to the sections which criticise our own work.

MM use a different approach from ourselves to the study of rotation and it is, we feel, largely due to this that most of the misconceptions embodied in their criticisms arise. They use a particular metric

$$ds^2 = dt^2(1-\omega^2 r^2/c^2) - c^{-2}(dr^2 + r^2\, d\theta^2 + 2\omega r^2\, d\theta\, dt) \tag{1}$$

for the rotating system and derive all of their results from this. This metric is derived by means of a Galilean transformation in which r and t are the same as in the inertial system of the centre but θ is related to the angle in the inertial system of the centre by the subtraction of ωt. It is not generally accepted that this is the correct metric to apply to a rotating system, although it may be applied for small values of $\omega r/c$ (see, for example, Atwater 1974). The use of the metric in equation (1) results in solutions for the geodesics with local particle velocities greater than c. This we find to be completely unacceptable. In contrast to this approach we have been concerned to attempt to produce a description of the rotating system from the point of view of measurements based on the physical information *directly* available to an observer fixed at any point on a rotating system which need not necessarily be a disc.

We reply to the major criticism by MM as follows:

In their abstract they state that we 'claim ... a contraction of length in the radial direction' and that 'it is shown that the claim is without foundation'. Our claim is, however, that a *radar* measurement of the radial distance, made by a rotating observer, shows a contraction (Jennison 1964), and we have also shown that the parallax distance to the centre shows the same contraction (Jennison and Ashworth 1976). We have clearly stated (Ashworth and Jennison 1976) that other techniques of measuring a radial distance can be used and that these may exhibit no contraction. For example we have shown that an observer who measures the radius of a disc by actually walking towards the centre will find that he has walked a distance equal to the inertial radius. This observer, however, changes his pseudo-gravitational potential with every step he takes and is therefore in no way equivalent to the observer who measures entirely from a fixed distance on a rotating ring. The measurement of length in special relativity is based on the standard procedure of synchronisation introduced by Einstein and, in

0305-4770/78/0011-9259$01.00

Euclidean flat space, distance and length are virtually synonymous. In non-Euclidean space the measurement of length can only be achieved as a close approximation by integrating vanishingly small elements (e.g. the 'integrated piecemeal radius' used in Ashworth and Jennison 1976). The measurement of *distance*, however, depends on the type of distance under discussion, e.g. radar distance, parallax distance, luminosity distance, integrated piecemeal distance etc. The first three of these do not depend upon the integration of the elements in non-Euclidean space but upon the relevant parameters observed at a single ring of constant radius. MM, on the other hand, appear to consider that the integrated piecemeal length is the only meaningful measurement of distance and that all other measurements have no practical significance.

MM consider that the experiment of Davies and Jennison (1975) does not prove a radial contraction and they attempt to show this by criticising the assumptions clearly stated in that paper, e.g. that the speed of light is locally invariant to an observer measuring it on the rotating system. In strict terms the experiment shows that a signal sent from the centre to the circumference of a rotating system where it is received and re-transmitted exhibits no frequency shift when it is ultimately received back at the centre. The result of the experiment was used together with the stated assumptions to show that an observer at the circumference would measure a radius of $(1-\omega^2r^2/c^2)^{1/2}$ times the inertial radius, r, if he used a radar measurement. The argument was based on the use of local clocks to measure the time of travel of a light ray from the observer, to some remote point, and back again. It is here that the disagreement with MM arises. We maintain that an observer on a rotating ring can make a measurement of his radar distance or parallax distance to the centre by using the appropriate technique. Thus the radar distance is expressed simply in local terms from the proper time interval and the invariant local value of the velocity of light, c. The observer will measure the local velocity of light and determine the value c, just as on the rotating earth. If he uses this together with the time of travel t' on his local clock then he must arrive at a value of $ct'/2$ for the radar range. We were not the first to define radar distance in this way; it was certainly in use fifty years ago, but we fully accept it as a useful and meaningful concept. Both radar and parallax distance may be applied on the macroscale of astronomy and the microscale of particle physics where, in both cases, integrated lengths are unavailable. They provide the local observer's effective Euclidean distance to remote regions by the extrapolation of entirely local units.

MM also criticise us for interpreting the coordinate (r, θ, t) as 'pertaining somehow to one particular observer'. In the analysis by Davies (1976) the coordinate (r, θ, t) certainly pertained only to measurements made by an observer at the centre. In their conclusion, however, MM also quote this as an objection to the paper by Davies and Jennison (1975). Davies and Jennison did not invoke this concept in determining the radial contraction. The observer may be anywhere on any rotating system, such as a simple particle in orbit. Our analyses and our measurements of the radial distance associated with a single observer are not restricted to measurements on a well ordered rotating disc, the radar and triangulation measurements would apply if the intervening material were in any state of motion whatsoever. We claim that the radar and triangulation distance to the centre is not affected by this motion whereas MM stipulate a specific ordered motion for the integration of the element of length. This integration could not be performed, for example, in a system with discontinuities such as a set of concentric, alternately contra-rotating, rings but radar and triangulation (parallax) distances may still be measured. It is of course possible to describe a disc in terms of any coordinates one chooses. MM state that their coordinates (r, θ, t) have a 'perfectly objective, observer free meaning' but these coordinates are only proper for an observer at the central singularity, i.e. they may be only *directly* related to measurements made at the central singularity. We describe events in terms of proper, i.e. local, measurements, directly available to the rotating observer wherever he may be situated on the system. Like MM, we have discussed two useful representations of geodesics upon the disc (Ashworth and Jennison 1976, Ashworth and Davies 1977), but, contrary to the suggestion made by MM, we have never claimed that one of them is 'correct'.

The circular arc solutions for ray paths have rather more significance than that credited to them by MM. If one incorporates the radial contraction discarded by MM, the circular arcs inscribed on a flat chart provide a universal diagram in which the radial distance to the centre is read off the diagram as the relevant *radar* or *parallax* distance of each ring from the centre, the diagram is then a powerful navigational aid which correctly accounts for the local observations of any single observer wherever he may be situated on the system, for the scale of the whole diagram shrinks as he moves into the outer parts (see figure 1).

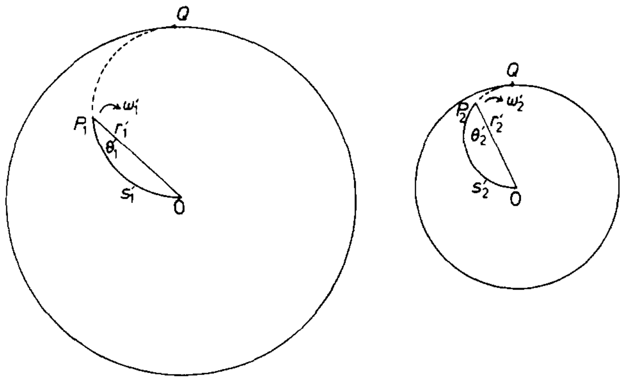

Figure 1. Universal chart based on $s' = (c/\omega')\sin^{-1}(\omega' r'/c)$ (Jennison 1964) where primes refer to measurements made by the observer at P. It is remarkable that this flat chart with its appropriate scale factor $(r'/r = \omega/\omega' = (1-\omega^2 r^2/c^2)^{1/2})$ is universally applicable to an observer at any rotating point P. The local angular velocity ω' may be obtained by direct observation of the distant stars or of a local free compass, r' is the radar or parallax distance to the centre and θ' is the angle between a local plumb line and the line of sight to the centre. The circular arc s' is purely diagrammatic and is not a measured distance. Note that the scale of the whole diagram varies with the position of P. For $P \to Q$, $\omega' r' \to c$, $\theta' \to \pi/2$, $r' \to 0$ and the diagram vanishes.

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